OPTIMIZATION OF RELIABILITY VERIFICATION TEST STRATEGIES

SUMMARY
Reliability verification can be time consuming and expensive, especially for highly reliable products. This paper presents a method for minimizing the cost of reliability verification tests. The methodology presented takes advantage of economies gained through bayesian testing when prior knowledge of the shape parameter of the Weibull distribution is known.

KEY WORDS
Reliability, Automotive, Economics, Optimization, Testing

1.0 INTRODUCTION
Reliability verification testing is almost always included as a quality specification for new products. A typical example in the automotive industry is to demonstrate 95% reliability at 100,000 miles with 90% confidence. Obviously, in this case some type of accelerated testing is required. It is infeasible to accumulate 100,000 miles or more on an automobile at a normal rate; the design would be 10 years out of date before testing is completed.

Even with acceleration, reliability verification can be time consuming and expensive, especially for highly reliable products. The costs of verification testing include:

a) prototype costs,
b) machine time costs,
c) labor costs, and
d) development time costs.

Prototype costs are simply the cost to produce the units to be tested. In some cases, prototypes are hand made by engineers, resulting in high costs. In other cases, the units to be tested are selected from the initial batch of the production process, which results in a lower cost.

The machine time cost is the cost of operating the test machinery. If testing is conducted by an outside laboratory, machine time cost is equal to the charge per unit time. If in house equipment is used, the cost is more difficult to compute. Factors such as energy, maintenance, depreciation, and utilization should be included. In some cases, equipment will have to be purchased to complete testing on schedule.

Labor costs consist of the personnel costs required to conduct the testing. Technicians or engineers are usually required to monitor the equipment and units being tested, and to record the results of the testing. This cost is usually in the form of a step function. For example, one technician may be sufficient for testing up to 8 units, but if 9 units are tested, 2 technicians are required.

Development time costs are the most difficult of the costs to quantify. Recent literature emphasizes the importance of reducing product development cycle times, and getting new products to market quickly. Reliability verification testing often represents a substantial portion of the development time cycle. Development time cost represents the cost per unit time of delaying a product's release. For example, management may believe that it is worth $2 million if a product can be released 6 months early. On the other hand, management may feel that delaying a product's release by 3 months would cost the company $1 million in revenue.

One of the first decisions to be made when designing a reliability verification test is to determine how many units to test. If many units are tested, the duration of the test will be short. With this approach, prototype costs will be high, and labor and development time costs will be low. If few units are tested, the duration of the test will be longer: prototype costs will be low, but labor and development time costs will be high.

Following is a methodology to minimize the cost of a reliability verification test. The methodology presented determines how many units should be tested, and takes advantage of economies gained through bayesian testing when prior knowledge of the shape parameter of the Weibull distribution is known.

2.0 TESTING STATISTICS

The Weibull reliability distribution is

$$R(t) = e^{\left[-\left(\frac{t}{\theta}\right)^{\beta}\right]} \tag{1}$$

where β is the shape parameter, and
θ is the scale parameter.

To estimate these parameters and compute confidence limits requires at least two failures (Dodson, 1995). But when conducting reliability verification tests, it is often infeasible to test until two failures occur. Often, no failures are encountered during verification testing.

Weibull parameters can be computed with few or no failures if the shape parameter is known. Satisfactory estimates of the shape parameter are available from many sources. Nelson (1985) states that the shape parameter estimated from fatigue specimens of an alloy used to make turbine disks may be used to as an estimate of the shape parameter for the failure distribution of the disks themselves. Another approach is to use the shape parameter from a previous design. For example, field data may be used estimate the shape parameter of the failure distribution of a hydraulic control unit. It is reasonable to expect that the shape parameter of the failure distribution for the next generation design is equal to that of the previous design, or at least very close. Also, there is a body of evidence that provides estimated shape parameters for the failure distribution for specific items. For example, it is generally acknowledged that the shape parameter for the life of bearings is 1.5.

Assuming the value of the shape parameter is equal to β, the maximum likelihood estimate of the scale parameter is

$$\hat{\theta} = \left(\frac{\sum_{i=1}^{n} t_i^{\beta}}{r} \right)^{1/\beta} \tag{2}$$

where, t_i is the time to fail or the time of censoring for the *i*th unit,
n is the number of units tested, and
r is the number of failures.

The lower confidence limit for the estimated scale parameter is

$$\theta_{L,\alpha} = \left(\frac{2\sum_{i=1}^{n} t_i^{\beta}}{\chi^2_{\alpha,2r+2}} \right)^{1/\beta} \tag{3}$$

where α is the level of significance (α = 0.1 for 90% confidence), and
$\chi^2_{\alpha,\nu}$ is the critical value of the chi-square distribution with significance α, and ν degrees of freedom

This expression assumes testing is discontinued after a predetermined amount of time. If testing is discontinued after a predetermined number failures, the degrees of freedom for the chi-square statistic is 2*r*.

The estimated reliability is

$$\hat{R}(t) = e^{\left[-\left(\frac{t}{\hat{\theta}}\right)^{\beta}\right]} \tag{4}$$

The lower confidence limit for reliability is

$$R_{L,\alpha}(t) = e^{\left[-\left(\frac{t}{\theta_{L,\alpha}}\right)^{\beta}\right]} \tag{5}$$

3.0 EFFECTS OF THE SHAPE PARAMETER ON TEST DURATION

From the equations given in the previous section, it can be seen that the test time required to demonstrate a specified reliability, at time *t*, with confidence 1-α, and assuming no units fail, is

$$T = \theta\left(\frac{-\ln\alpha}{n}\right)^{1/\beta} \tag{6}$$

where θ is the characteristic life corresponding to the required reliability at time = *t*, and is

$$\theta = \frac{t}{\left[-\ln R(t)\right]^{1/\beta}} \tag{7}$$

From Equation 6, it is obvious that the duration of the test time is not directly inversely proportional to the number of units tested. Figure 1 shows the relationship between test duration and sample size for a verification test requiring 95% reliability with 90% confidence at 100,000 miles, assuming a shape parameter of 3.

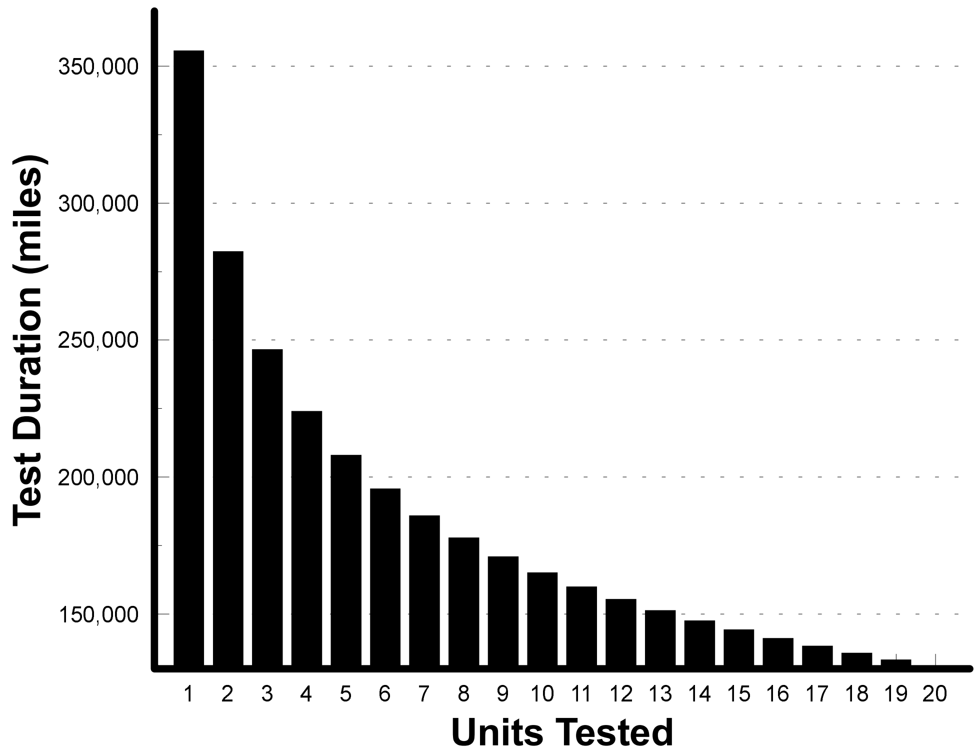

Figure 1. Test duration versus number of units tested to ensure 95% reliability with 90% confidence at 100,000 miles, given a shape parameter of 3, and no failures.

From Figure 1, it can be seen that the total time on test (test duration X number of units tested), is increasing with the number of units tested. This is shown in Figure 2.

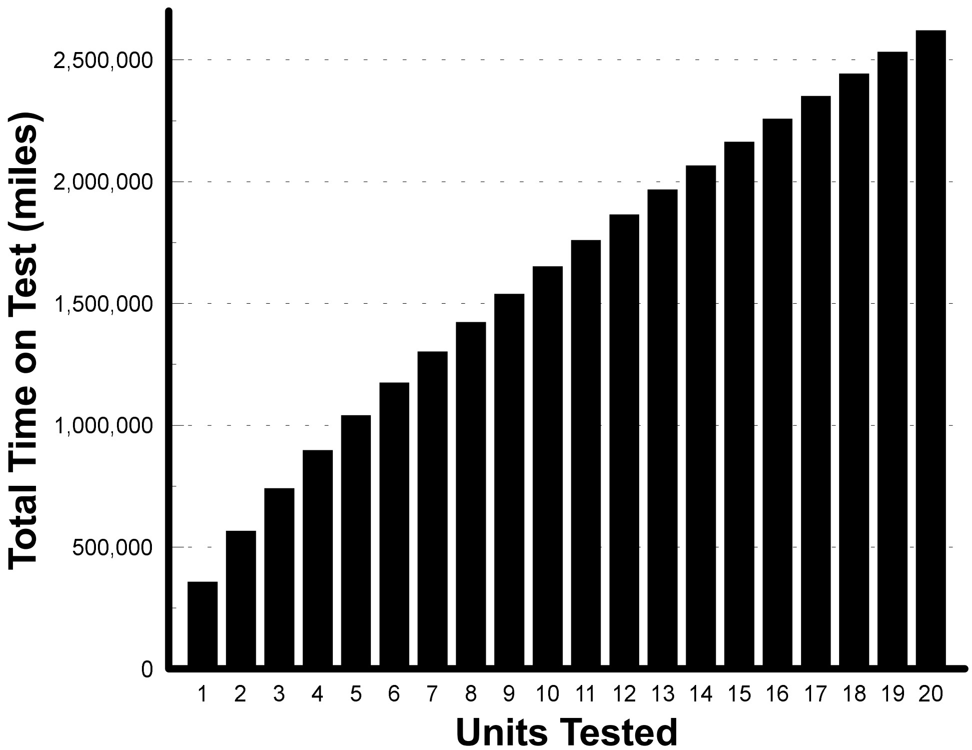

Figure 2. Total time on test versus number of units tested to ensure 95% reliability with 90% confidence at 100,000 miles, given a shape parameter of 3, and no failures.

The behavior displayed in Figures 1 and 2 is unique to the case of a shape parameter greater than 1.0. If the shape parameter is equal to 1, the total time on test required to achieve a specified reliability with a given confidence is independent of the number of units tested. If the shape parameter is less than 1, the total time on test required to achieve a specified reliability with a given confidence decreases as the of the number of units tested increases.

4.0 DETERMINING ECONOMICALLY OPTIMUM RELIABILITY VERIFICATION PLANS

The system cost of a reliability verification test is

$$C_s = T\left(C_l + C_d + C_m\right) + nC_p \tag{8}$$

where C_l is the labor cost per unit time,
C_d is the development time cost per unit time,
C_m is the machine time cost per unit time per machine,
C_p is the cost of prototypes per unit,
n is the number of units tested, and
T is the duration of the verification test.

When designing a reliability verification test, the major decisions are how long to conduct the test and how many units to test, to achieve the parameters of the test – specified reliability at the required time, with the desired confidence. Given a required reliability, R, at time T, with a confidence level of
$1-\alpha$, and assuming no failures occur during testing; the optimum number of units to test is found by minimizing Equation 8 in terms of n. This is easily done graphically. Simply determine the required testing duration from Equation 6, and graph Equation 8 with respect to n.
This relationship can be exploited to reduce the system cost of reliability verification testing.

Example

An accelerated test is used to simulate miles on an automobile. The machine cost, C_m , is $0.5 per simulated mile, the labor cost, C_l , is $0.45 per simulated mile, and the development time cost, C_d , is $0.5 per simulated mile. From prior testing with a similar design, it is known that the Weibull distribution adequately models the time to fail with a shape parameter of 3.0. The customer requires 95% reliability at 100,000 miles with 90% confidence. With a prototype cost, C_p , of $20,000 per unit, how many units should be tested and what is the duration of the test, assuming no failures occur during testing?

Solution

First, determine the required value of the scale parameter for 95% reliability at 100,000 miles using Equation 7. This value, 269,141 miles, is then input into Equation 6 for several values of *n*, and the minimum is determined graphically. Figure 3 displays the system cost for the verification test described above. The optimum test plan is to test 5 units for an equivalent of 207,840 miles.

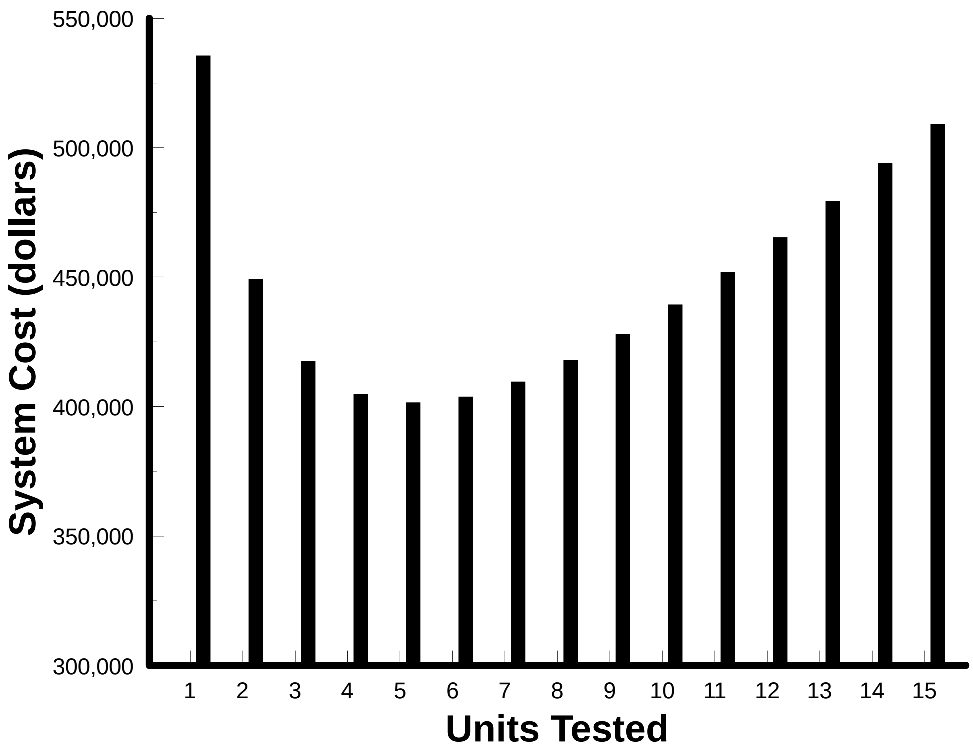

Figure 3. Graphical optimization of reliability verification test parameters.

Note that taking advantage of a known shape parameter in this example produces a tremendous savings. If knowledge of the shape parameter is ignored, conventional binomial testing requires 45 units be tested for an equivalent of 100,000 miles. The system cost savings is 62%, and the total time on test savings is 77%.

5.0 LIMITATIONS

An obvious limit of the methodology presented is that failures will be encountered during testing in some cases, especially when testing extends a great deal beyond the required reliability. If failures are encountered, simply replace the failed units and continue testing. Equations 3 and 5 can be used to determine confidence limits for reliability when failures are encountered. Also, if failures are encountered, the test plan should still be optimum, or very near optimum. Given the product being tested meets the reliability requirements, failures will not be encountered unless the optimum test plan calls for a lengthy test duration. But the optimum test plan will only require a lengthy duration if the number of units tested is small. This situation arises when the prototype cost is high, in which case, it is desirable to test as few units as possible, which would indicate that allowing units to fail during testing is desirable.

The example problem did not include a step function for the cost of machine time. In reality, a test machine will have a fixed capacity, and the machine cost will be proportional to the number of machines required. For example, the machine time cost may be $100 per unit time for testing up to 4 units, $200 per unit time for testing 5 to 8 units, $300 per unit time for testing 9 to 12 units, and so on. This step function for machine time cost is easily factored into the methodology above.

6.0 REFERENCES

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