Gage Bias

Bias is the difference between the output of the measurement system and the true value, and is often referred to as accuracy. The concept of bias is shown statistically in Figure 1. Figure 1 demonstrates negative bias; the gage underestimates. If the gage overestimates, the bias is positive.

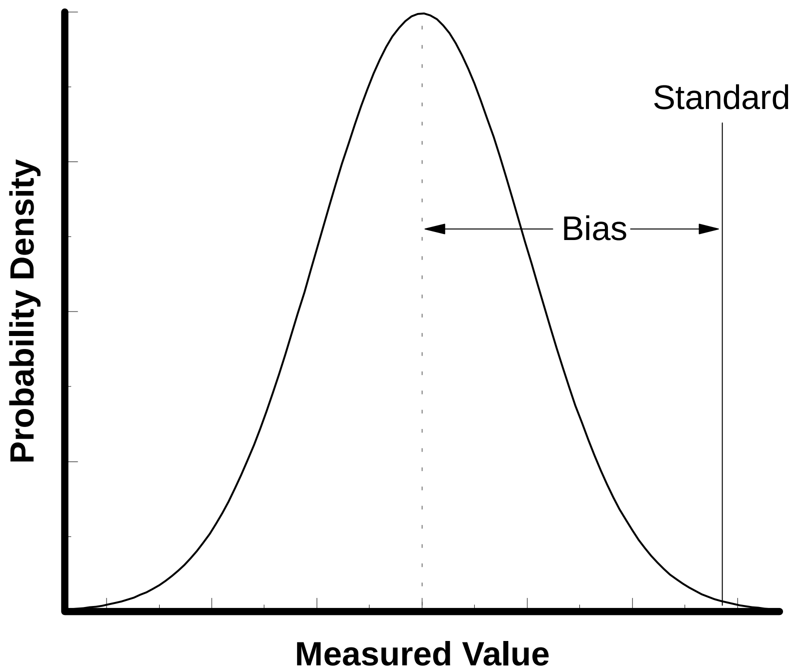

Figure 1. Example of bias in a pressure gage.

Bias is computed from the expression

$$\text{Bias} = \frac{\sum_{i=1}^{n} x_i}{n} - \tau \qquad 1$$

where n is the number of times the standard is measured,
x_i is the ith measurement, and
τ is the value of the standard.

Bias is usually reported as a percent of process variation or as a percent of tolerance. Given a process standard deviation of σ, bias as a percent of process variation (6σ) is

$$\text{Bias(Process Variation)} = 100\left[\frac{\dfrac{\sum_{i=1}^{n} x_i}{n} - \tau}{6\sigma}\right] \qquad 2$$

Given a tolerance (Upper specification - Lower specification) of *T*, bias as a percent of process tolerance is

$$\text{Bias(Tolerance)} = 100\left[\frac{\dfrac{\sum_{i=1}^{n} x_i}{n} - \tau}{T}\right] \qquad 3$$

To perform a bias study, a reference standard with a known value is necessary. Measure the part on the gage being tested a minimum of 10 times, and preferably more than 30 times. The greater the number of measurements, the greater the accuracy. In virtually every case, there will be bias, however, the bias may not be statistically significant. The significance is tested using the *t*-distribution, and increasing the number of measurements increases the discriminatory power of the *t*-test.

A gage is biased if the average measure of an item with a known value is significantly different than the known value. The significance of the bias is determined using a *t*-test. The test hypotheses are

$$\text{Ho}: \mu = \tau$$
$$\text{H}_1: \mu \neq \tau$$

The test statistic is

$$t = \frac{\overline{X} - \mu}{s / \sqrt{n}} \qquad 4$$

where $\overline{X}$ is the average of the measurements,
μ is the known measure of the standard,
s is the sample standard deviation of the measurements, and
n is the number of measurements taken.

For a 2-tailed test, the critical values of the *t*-distribution are

$t_{\alpha/2,n-1}$ and
$-t_{\alpha/2,n-1}$

where α is the level of significance chosen for the test.

Using a *t*-test assumes the measured values are normally distributed. This is not a critical assumption unless *n* is small because the *t*-test is testing the population average, and the distribution of the average tends to normal as *n* increases regardless of the distribution of the individuals. In most cases *n* = 10 is sufficient, even in cases where the distribution of the individuals greatly differs from normal, a sample size of 30 is usually sufficient.

Example

A block is known to weigh 100.3 pounds. This block is measured 30 times on a scale, and the resulting measures in pounds are given Table 1. The process standard deviation is 0.32, and the process tolerance is 7.4 (Upper Specification - Lower Specification).

Table 1. Bias example data.

100.1	100.6	101.5	101.6	101.8
101.9	100.2	100.8	101.3	100.0
101.0	101.1	102.0	100.5	100.7
100.2	101.6	100.9	100.6	100.4
100.6	100.8	100.5	101.0	100.0
101.5	101.4	101.3	100.3	101.3

The average weight is 100.92. The bias of the scale is

$$\text{Bias} = 100.92 - 100.3 = 0.62$$

Is this true bias, or is it due to the random error? Eleven of the 30 measurements underestimated the known weight of 100.3. To test for significance, compute samples standard deviation of the 30 values, and compute the *t*-statistic. The sample standard deviation is, s = 0.587, and the *t*-statistic is

$$t = \frac{\overline{X} - \mu}{s / \sqrt{n}} = \frac{100.92 - 100.3}{0.587 / \sqrt{30}} = 5.78$$

Since the bias appears to be positive, a 1-tailed test will be used. Using a significance level of $\alpha = 0.05$, with 29 degrees of freedom (30 -1), from Appendix C , the critical value of the *t*-distribution is

$$t_{0.05,29} = 1.699$$

There is no need to lookup statistical values in tables. The critical value of the t-distribution can be found using Lotus 123 with the formula @TDIST(0.05,29,1,1). The equivalent formula in Microsoft Excel is TINV(0.05/2,29). Excel does not provide a one-tailed function, so the significance must be divided by 2.

Since the computed *t*-statistic exceeds the critical value, the bias is significant, and the difference from the true value is not due to random error. If possible, the scale should be adjusted to yield a bias of zero.

The bias as a percent of process variation (6 standard deviations) is

$$\text{Bias(Process Variation)} = 100\left[\frac{100.92 - 100.3}{6(0.32)}\right] = 32.3\%$$

The bias as a percent of tolerance is

$$\text{Bias(Tolerance)} = 100\left[\frac{100.92 - 100.3}{7.4}\right] = 8.4\%$$