USING STATISTICS TO SCHEDULE MAINTENANCE

INTRODUCTION

Maintenance scheduling does not have to be based on "expert opinion". By carefully recording failure data, or using failure data from manufacturers, maintenance schedules can be economically optimized using statistical methods. Three types of maintenance will be considered

1) Preventive maintenance,
2) Inspections, and
3) Predictive maintenance

Preventive maintenance is the standard "PM". A PM is performed to prevent failures due to wear. Examples are changing hoses, changing belts, routine cleaning, etc. Inspections are used to reduce the impact of failures that are not catastrophic. Consider the human body. A cancer inspection has a cost (money, time, pain and embarrassment), but the damage created by the cancer increases with time if not treated (the cost of a failure is proportional to time). Predictive maintenance is used to prevent failures by detecting some type of warning, such as, increased vibration, increased particle count in oil, or increased temperature.

CENSORED VERSUS COMPLETE DATA

If ten items are tested until all ten fail, this is a complete data set. If the test is ended before all ten items fail, the items that did not fail are "censored."

Consider the data in the table below. Eight items were placed on test stands; three of the items failed, and five of the items were removed from testing without failing.

Example of Censored Data

30	60 +
40	60 +
50	60 +
60 +	60 +

Obviously, the sample average and the sample standard deviation for the three failed items cannot be used to estimate the parameters of the normal distribution in this case. The sample average is (30+40+50)/3 = 40. The time to fail for each of the remaining five items is greater than 60; the true average is considerably greater than 40.

The data in the table above are **right** censored. An item is censored on the right if the failure time is not known, but it is known that the item survived to a known time without failure. If an item is known to be in a failed condition at a specific time, but the exact failure time is not known, this is **left censoring**

Single censoring occurs when there is only one censoring point. If 100 transistors are placed on test stands and the test is terminated after 1000 hours, there is a single censoring point at 1000 hours. If 20 transistors were removed without failure after 1000 hours of testing and another 15 transistors were removed without failure after 1200 hours of testing, there are two censoring points, and the resulting data are **multiply censored**. If exact failure times are not known, but the numbers of failures in a time interval are recorded, this is **interval or grouped data**.

THE WEIBULL DISTRIBUTION

The Weibull distribution is a continuous distribution that was publicized by Waloddi Weibull in 1951. Although initially met with skepticism, it has become widely used, especially in the reliability field. The Weibull distribution's popularity resulted from its ability to be used with small sample sizes and its flexibility. In addition to being the most useful density function for reliability calculations, analysis of the Weibull distribution provides the information needed for troubleshooting, classifying failure types, scheduling preventive maintenance and scheduling inspections. The Weibull probability density function is

$$f(x) = \frac{\beta(x-\delta)^{\beta-1}}{\theta^{\beta}} \exp\left[-\left(\frac{x-\delta}{\theta}\right)^{\beta}\right], \; x \geq 0 \tag{1}$$

where β = the shape parameter,
θ = the scale parameter, and
δ = the location parameter.

Beta θ, and δ are continuous. The acceptable ranges for these variables are
$0 < \beta < \infty$,
$0 < \theta < \infty$, and
$-\infty < \delta < \infty$.

The estimation of these parameters is not straightforward, and special techniques such as probability plotting, hazard plotting, or maximum likelihood estimation are required.

EFFECTS OF THE SHAPE PARAMETER

By altering the shape parameter, β, the Weibull probability density function takes a variety of shapes. This is demonstrated in Figure 1.

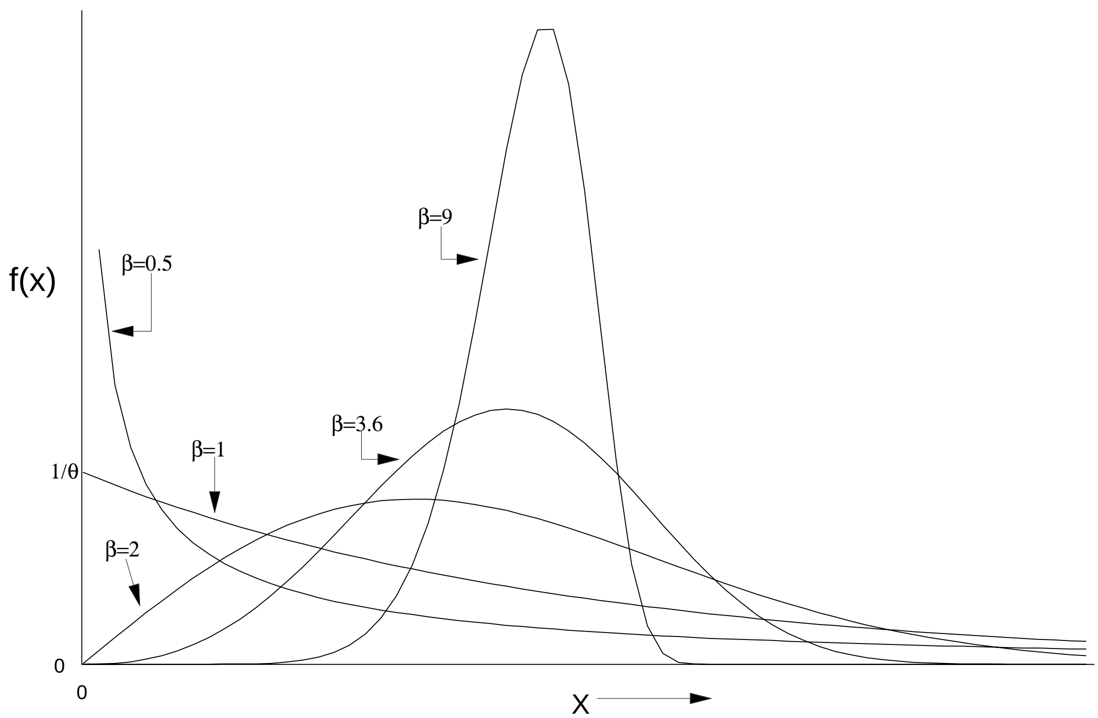

Figure 1. Weibull probability density functions.

Note that several of the probability density functions displayed in Figure 1 look familiar. The Weibull distribution can be used in a wide variety of situations and dependent on the value of β, is equal to or can approximate several other distributions. For example, if

β = 1, the Weibull distribution is identical to the exponential distribution,
β = 2, the Weibull distribution is identical to the Rayleigh distribution,
β = 2.5, the Weibull distribution approximates the lognormal distribution,
β = 3.6, the Weibull distribution approximates the normal distribution. and
β = 5, the Weibull distribution approximates the peaked normal distribution.

Because of this flexibility, there are few observed failure rates that cannot be accurately modeled by the Weibull distribution. Some specific cases are

- the breaking strength of components or the stress required to fatigue metals,
- the time to fail for electronic components,
- the time to fail for items that wear out, such as automobile tires, and
- systems that fail when the weakest component in the system fails.

METHODS OF PARAMETER ESTIMATION

The four most widely used methods of parameter estimation are:

1) Maximum Likelihood Estimation,
2) Moment Estimation,
3) Probability Plotting, and
4) Hazard Plotting.

MAXIMUM LIKELIHOOD ESTIMATION

Maximum likelihood is the most widely used method for generating estimators. It is based on the principle of determining the parameter(s) value(s) that maximize(s) the probability of obtaining the sample data.

The likelihood function for a given distribution is a representation of the probability that the sample data of obtaining the sample values. Let $x_1, x_2, \ldots, x_n$ be independent, random variables from the probability density function $f(x, \theta)$, where θ is the single distribution parameter. Then

$$L(x_1, x_2, \ldots, x_n; \theta) = f(x_1, \theta) f(x_2, \theta) \ldots f(x_n, \theta)$$

is the joint distribution of the random variables, or the **likelihood function**. The maximum likelihood estimate, $\hat{\theta}$, maximizes the likelihood function. This estimate is asymptotically normal. Often the natural logarithm of the likelihood function is maximized to simplify computations.

MOMENT ESTIMATION

Moment estimation is based on the concept of matching the moments of the sample data with the moments defined by the distribution of interest and its parameters. For example, when estimating the parameters of the two parameter Weibull distribution, the first and second moments from the sample data, the sample mean and the sample variance, would be equated to the expressions

$\mu = \theta \Gamma\left(1 + \frac{1}{\beta}\right)$ and

$$\sigma^2 = \theta^2\Gamma\left[\Gamma\left(1+\frac{1}{\beta}\right)-\Gamma^2\left(1+\frac{1}{\beta}\right)\right]$$

PROBABILITY PLOTTING

Probability plotting is a graphical method of parameter estimation. The cumulative distribution function is linearized, usually by a logarithmic transformation, and plotted. The slope and the intercept of the plot provide the information needed to estimate the parameters of the distribution of interest. The median rank is used to estimate the cumulative distribution function, and ranks at user input levels are used to provide confidence intervals for reliability. If manually constructing a probability plot, distribution specific hazard paper is required. By using probability paper, the failure times and cumulative distribution function estimates can be plotted directly. With the power of personal computers, specialized graph paper is no longer needed, as the necessary transformations can be made quickly and easily.

HAZARD PLOTTING

Hazard plotting is a graphical method of parameter estimation. The cumulative hazard function is linearized, usually by a logarithmic transformation, and plotted. The slope and the intercept of the plot provide the information needed to estimate the parameters of the distribution of interest. If manually constructing a hazard plot, distribution specific hazard paper is required. By using hazard paper, the failure times and cumulative hazard function estimates can be plotted directly. With the power of personal computers, specialized graph paper is no longer needed, as the necessary transformations can be made quickly and easily.

PREVENTIVE MAINTENANCE

In some cases, it is possible to prevent failures with preventive maintenance. The question is to determine if preventive maintenance is applicable, and if so, how often should it be scheduled. Referring to Figure 2, failures can be grouped into 3 categories based on the behavior of the failure rate. **Infant mortality** failures are characterized by a decreasing failure rate. The hazard function (failure rate) of the Weibull distribution is decreasing if the shape parameter, β, is less than 1.0. **Random** failures exhibit a constant failure rate; the shape parameter of the Weibull distribution is equal to 1.0. **Wear-out** failures have an increasing failure rate; the shape parameter of the Weibull distribution is greater than 1.0.

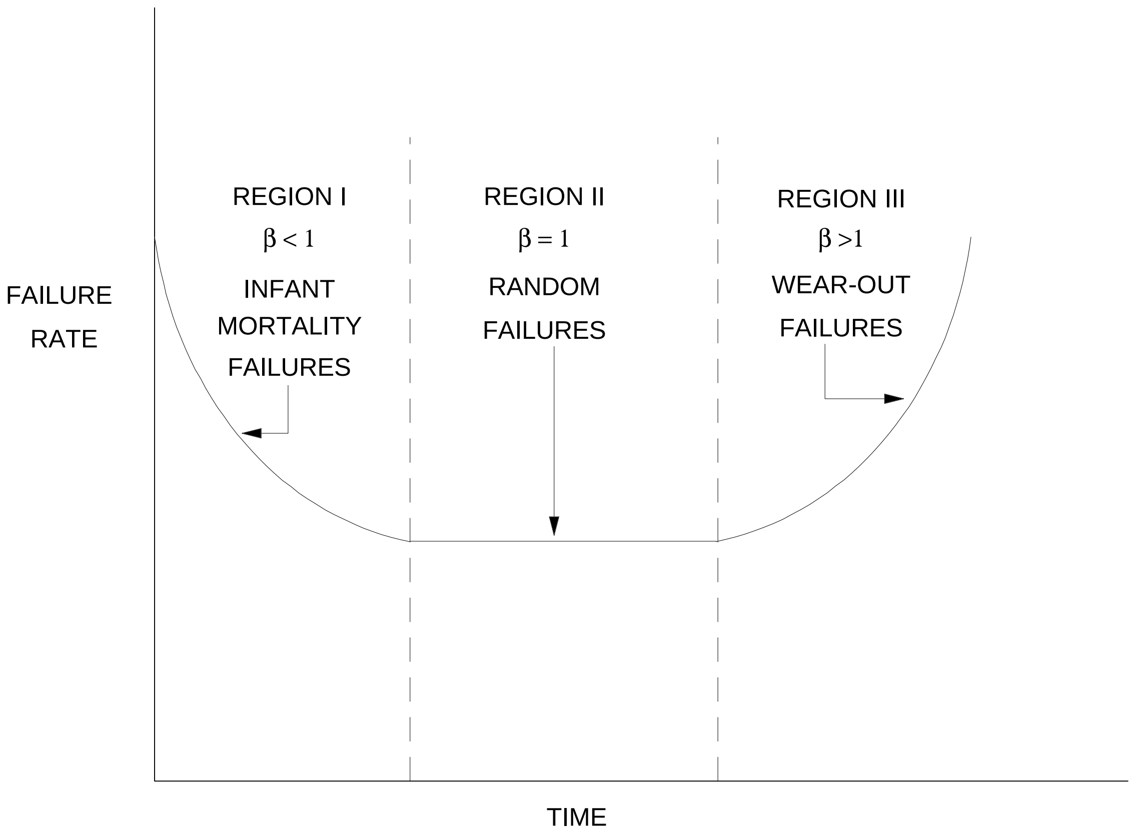

Figure 2. The bathtub curve.

Infant mortality failures are premature failures that can often be prevented by management. If infant mortality failures cannot be prevented, a burn-in procedure can be implemented to eliminate failures before the product is shipped. Preventive maintenance is not applicable for an item with a decreasing failure rate. Performing preventive maintenance restores the system to its initial state which has a higher failure rate; preventive maintenance increases the number of failures in this case.

Some causes of infant mortality failures are

- Improper use
- Inadequate materials
- Over-stressed components
- Improper setup
- Improper installation
- Poor quality control
- Power surges
- Handling damage

Random failures cannot be prevented with preventive maintenance. The failure rate is constant, so preventive maintenance has no affect on failures. Reliability can be increased by redesigning the item, or in some cases, by implementing an inspection program.

Wear-out failures can be prevented with preventive maintenance. The failure rate is increasing with time, so preventive maintenance restores the system to a state with a lower failure rate. The question is how often should preventive maintenance be scheduled.

The time to fail for an item is variable, and can be represented by a probability distribution, $f(x)$. Referring to Figure 3, the cost of failures per unit time decreases as preventive maintenance is done more often, but the cost of preventive maintenance per unit time increases. There exists a point where the total cost of failures and preventive maintenance per unit time is at a minimum; the optimum schedule for preventive maintenance.

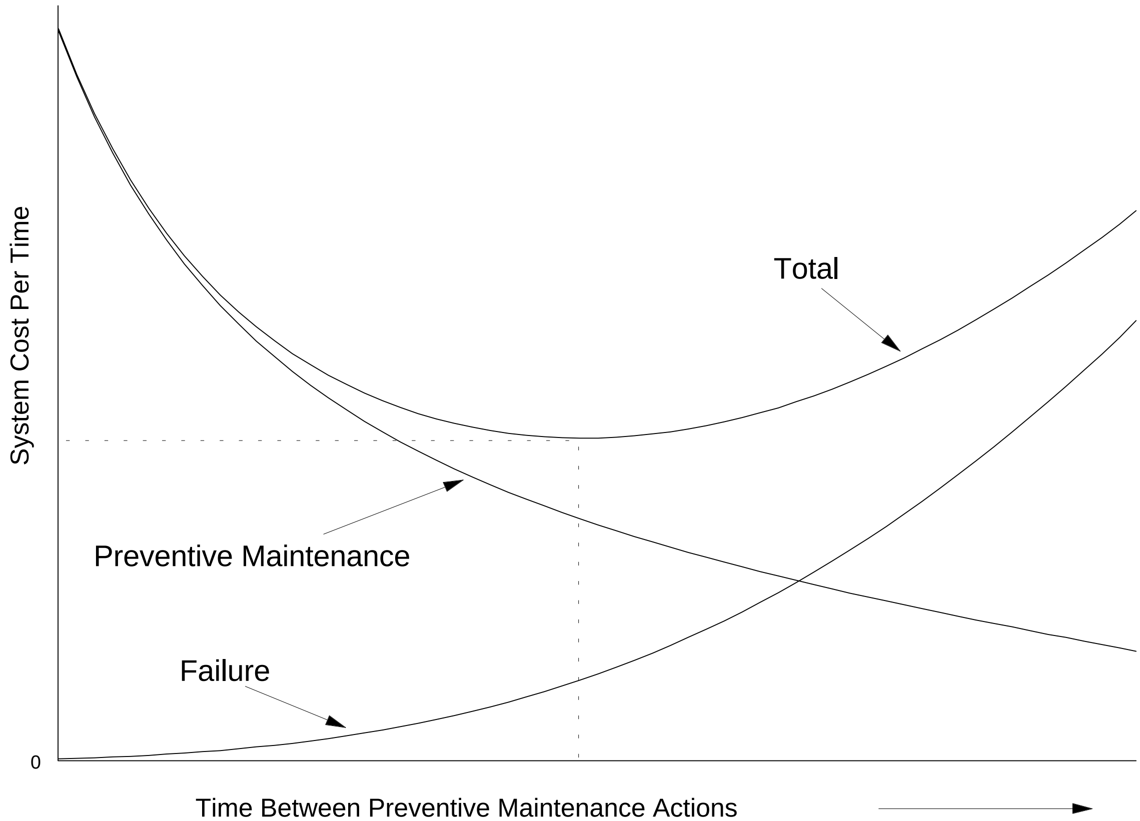

Figure 3. Optimum schedule for preventive maintenance.

The optimum time between maintenance actions is found by minimizing the total cost per unit time.

$$C_T = \frac{C_p \int_T^{\infty} f(t)dt + C_f \int_0^T f(t)dt}{T\int_T^{\infty} f(t)dt + \int_0^T tf(t)dt} \tag{2}$$

where C_p is the cost of preventive maintenance,
C_f is the cost of a failure, and
T is the time between preventive maintenance actions.

Minimizing Equation 2 is tedious, and numerical routines are usually required. Dodson (1994), developed a tabular solution for this problem given the following assumptions.

1. The time to fail follows a Weibull distribution.
2. Preventive maintenance is performed on an item at time T at a cost of C_p.
3. If the item fails before time = T, a failure cost of C_f is incurred.
4. Each time preventive maintenance in performed, the item is returned to its initial state; that is, the item is "as good as new."

The optimum time between preventive maintenance actions is

$$T = m\theta + \delta \tag{3}$$

where m is a function of the ratio of the failure cost to the preventive maintenance cost and the value of the shape parameter, and is given in Table 1.

θ is the scale parameter of the Weibull distribution, and

δ is the location parameter of the Weibull distribution.

Table 0. Values of m.

	ß							
C_f/C_p	**1.5**	**2.0**	**2.5**	**3.0**	**4.0**	**5.0**	**7.0**	**10.0**
2.0	2.229	1.091	0.883	0.810	0.766	0.761	0.775	0.803
2.2	1.830	0.981	0.816	0.760	0.731	0.733	0.755	0.788
2.4	1.579	0.899	0.764	0.720	0.702	0.711	0.738	0.777
2.6	1.401	0.834	0.722	0.688	0.679	0.692	0.725	0.766
2.8	1.265	0.782	0.687	0.660	0.659	0.675	0.713	0.758
3.0	1.158	0.738	0.657	0.637	0.642	0.661	0.702	0.749
3.3	1.033	0.684	0.620	0.607	0.619	0.642	0.687	0.739
3.6	0.937	0.641	0.589	0.582	0.600	0.627	0.676	0.730
4.0	0.839	0.594	0.555	0.554	0.579	0.609	0.662	0.719
4.5	0.746	0.547	0.521	0.526	0.557	0.591	0.648	0.708
5	0.676	0.511	0.493	0.503	0.538	0.575	0.635	0.699
6	0.574	0.455	0.450	0.466	0.509	0.550	0.615	0.683
7	0.503	0.414	0.418	0.438	0.486	0.530	0.600	0.671
8	0.451	0.382	0.392	0.416	0.468	0.514	0.587	0.661
9	0.411	0.358	0.372	0.398	0.452	0.500	0.575	0.652
10	0.378	0.337	0.355	0.382	0.439	0.488	0.566	0.645
12	0.329	0.304	0.327	0.357	0.417	0.469	0.550	0.632
14	0.293	0.279	0.306	0.338	0.400	0.454	0.537	0.621
16	0.266	0.260	0.288	0.323	0.386	0.441	0.526	0.613
18	0.244	0.244	0.274	0.309	0.374	0.430	0.517	0.605
20	0.226	0.230	0.263	0.298	0.364	0.421	0.508	0.598
25	0.193	0.205	0.239	0.275	0.343	0.402	0.492	0.584
30	0.170	0.186	0.222	0.258	0.328	0.387	0.478	0.573
35	0.152	0.172	0.207	0.245	0.315	0.374	0.468	0.564
40	0.139	0.160	0.197	0.234	0.304	0.364	0.459	0.557
45	0.128	0.151	0.187	0.225	0.295	0.356	0.451	0.550
50	0.119	0.143	0.179	0.217	0.288	0.348	0.444	0.544
60	0.105	0.130	0.167	0.204	0.274	0.335	0.432	0.534
70	0.095	0.120	0.157	0.193	0.264	0.325	0.422	0.526
80	0.087	0.112	0.148	0.185	0.255	0.316	0.415	0.518
90	0.080	0.106	0.141	0.177	0.248	0.309	0.407	0.513
100	0.074	0.101	0.135	0.172	0.241	0.303	0.402	0.507
150	0.057	0.082	0.115	0.150	0.217	0.278	0.379	0.487
200	0.047	0.071	0.103	0.136	0.203	0.263	0.363	0.472
300	0.035	0.058	0.087	0.119	0.182	0.243	0.343	0.454
500	0.025	0.045	0.071	0.100	0.161	0.219	0.319	0.431
1000	0.016	0.032	0.054	0.079	0.135	0.190	0.288	0.403

Example 1

The cost of failure for an item is $1000. The cost of preventive maintenance for this item is $25. The following Weibull distribution parameters were determined from time to fail data: β =2.5, θ = 181 days, δ = 0. How often should preventive maintenance be done?

Solution

The ratio of failure cost to PM cost is

$$\frac{C_f}{C_p} = \frac{1000}{25} = 40$$

Entering Table 0 with this ratio and a shape parameter of 2.5, give 0.197 for the value of *m*.

A PM should be done every

$$T = (0.197)(181) + 0 = 35.657 \text{ days}$$

This is shown graphically in Figure 4.

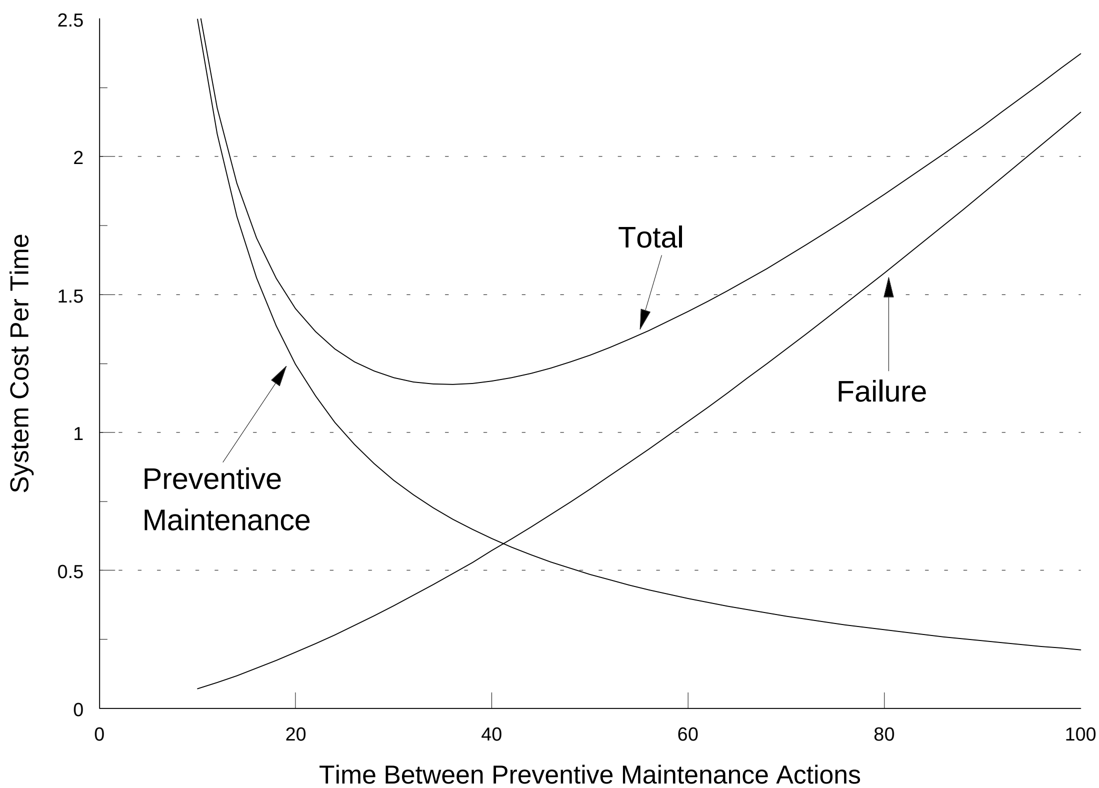

Figure 4. Solution to Example 1.

Example 2

Repeat Example 1 with the following Weibull distribution parameters: β =0.8, θ = 181 days, δ = 0.

Solution

When the shape parameter of the Weibull distribution is less than or equal to 1.0, the optimum solution is to do no preventive maintenance.

INSPECTIONS (CANCER SCREENING)

In some cases it is impossible to determine if a defect exists without inspection. This may be a defect causing scrap production or a defect causing machinery damage. The problem is to determine how often inspections should take place. There is an inspection cost, but the cost of an undetected defect in the process increases with time.

This is the same theory used to determine how often to screen people for cancer. A patient must visit the doctor to determine if there is a defect in the machinery (cancer in the patient). The longer the defect exists without detection, the higher the cost (the longer the cancer goes untreated, the more difficult it is to cure).

The assumptions for this routine are:

1) The time to fail follows a Weibull distribution.
2) The cost of a defect in the process increases linearly with time.
3) Inspections are 100% accurate at detecting a defect.

The output is an inspection schedule. The time between successive inspections will not be equal as in the case of preventive maintenance. If the failure rate is increasing (the shape parameter of the Weibull distribution is greater than 1), the time between successive inspections will decrease. This is similar to screening for cancer; the initial inspection may not take place until the person is 40 or 50 years old, and then as the person continues to age inspections occur more and more frequently. If the failure rate is decreasing (the shape parameter of the Weibull distribution is less than 1), the time between successive inspections will increase. If the failure rate is constant (the shape parameter of the Weibull distribution is equal to 1), the time between successive inspections will be constant.

The optimum length of time until the next inspection should take place, Δ, is found by minimizing the steady state cost for a given time period $\left(t_i, t_i + \Delta\right)$ in terms of Δ. The steady state cost is

$$C_s = C_i + \left[\int_{t_i}^{t_i+\Delta} f(x)dx\right]\left[\left(t_i + \Delta\right) - \left(\frac{\int_{t_i}^{t_i+\Delta} xf(x)dx}{\int_{t_i}^{t_i+\Delta} f(x)dx}\right)\right]C_d \tag{4}$$

where C_i is the cost of an inspection,

C_d is the cost of a defect per unit time, and

$f(x)$ is the probability density function representing the time between defects.

Example 3

In rolling mills, rolls are changed periodically. In some cases, they are changed because the product being rolled is changed, and in some cases they develop roll marks. If a roll mark develops, every coil rolled is scrap. The only way to detect the mark is by inspection. This inspection causes scrap and downtime. Using the failure data in the table below, assuming an inspection costs $5000, and a scrap coil costs $8000, determine when the next 3 inspections should take place after new rolls are installed.

Table 2. Coils until roll mark develops *.

1	2	8	20 +	20 +	20 +	20 +
1	2	8	20 +	20 +	20 +	20 +
1	2	8	20 +	20 +	20 +	20 +
1	2	9	20 +	20 +	20 +	20 +
1	2	10	20 +	20 +	20 +	20 +
1	4	12	20 +	20 +	20 +	20 +
1	5	17	20 +	20 +	20 +	20 +
1	5	18	20 +	20 +	20 +	20 +
1	5	20 +	20 +	20 +	20 +	20 +
2	8	20 +	20 +	20 +	20 +	20 +

* A + following an entry indicates that rolls were changed without failure.

SOLUTION

The MLE parameter estimates are

β (shape parameter) = 0.618
θ (scale parameter) = 55.49

The optimum time for the first 3 inspections are

1st inspection 7.115 coils $\rightarrow$ 7 coils
2nd inspection 15.56 coils $\rightarrow$ 15 coils
3rd inspection 24.86 coils $\rightarrow$ 25 coils

Note that the time between each successive inspection increases.

PREDICTIVE MAINTENANCE

Predictive maintenance tools, such as vibration analysis and oil analysis, are increasingly being used to prevent machinery failures and thus reduce operating expenses. An issue that has not received enough attention is how often to schedule inspections. Referring to Figure 5, if inspections are done too frequently, inspection costs are greater than savings costs. If inspections are scheduled too far apart, the opportunity savings of preventing failures is being underutilized.

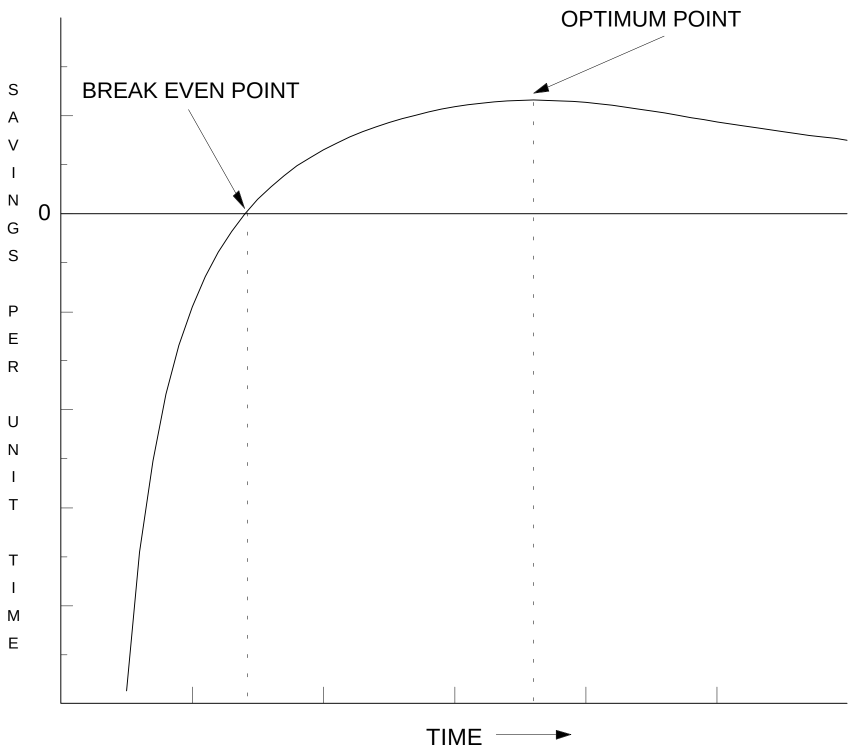

Figure 5. Optimum predictive maintenance schedule.

Referring to Figure 6, there are three possibilities when an inspection takes place:

1) No problems are found **-** zone A
2) An impending failure is discovered **-** zone B
3) The machinery failed before an inspection took place **-** zone C.

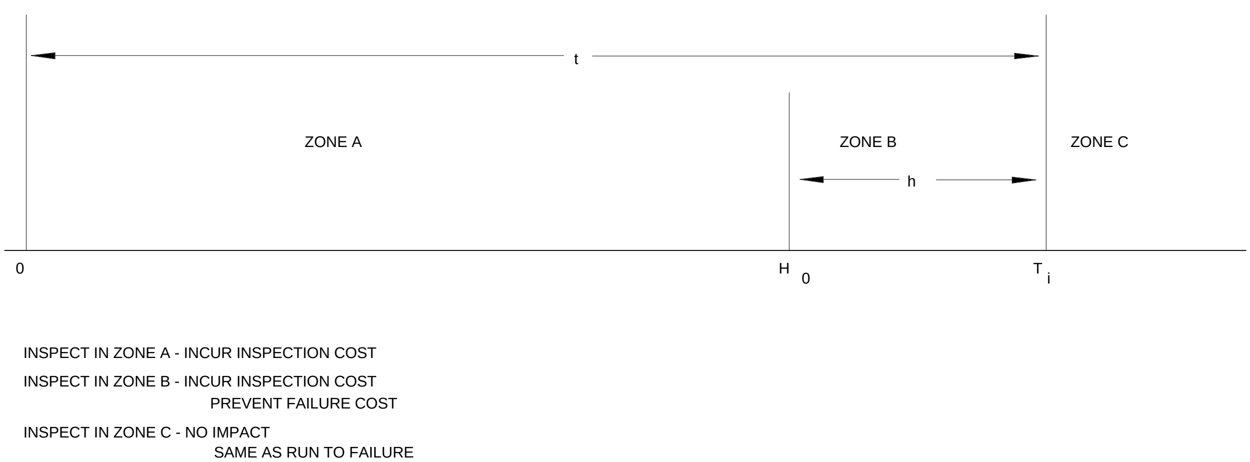

Figure 6. Possibilities when an inspection takes place.

Zone B, when an impending failure is discovered, will be referred to as the lapse zone. The length of this zone is variable and is represented by *h(x)*. The time to fail, which is also variable, is defined as *f(t)*. The length of time before an impending failure may be discovered is

$$g(z) = f(t) - h(x) \tag{5}$$

If an inspection takes place in zone A, the cost of an inspection has been incurred and no savings have been realized. If an inspection takes place in zone B, the cost of an inspection has been incurred and the cost of a failure has been prevented. An inspection cannot take place in zone C, the system has failed. Using the following definitions

A = cost per inspection
B = cost per failure
T_i = the time of inspection i
M = the average time to fail given a failure and survival to the previous inspection time.

The total savings per unit time is

$$S = \frac{P(t < H_0)(-A) + P(H_0 < t < T_i)(B - A)}{P(t > T_i)M + P(t < T_i)T_i} \tag{6}$$

$P(t<H_0)$ is the probability that the inspection takes place in zone A, $P(H_0<t<T_i)$ is the probability the inspection takes place in zone B. $P(t>Ti)$ is the probability of a failure before an inspection takes place (zone C). The problem is to determine the values of T_i (T_1 being the time for the first inspection, T_2 being the time of the second inspection, etc.) that minimizes the total savings per unit time. It is assumed that the system is restored to a state that is "as good as new" following a failure or the detection of an impending failure.

Determining the probabilities above can be difficult if $f(t)$ and $h(x)$ are not normally distributed. However, making this assumption would limit the applications of a solution since the most common distribution for failure times is the Weibull distribution. A solution for T_1 can be found with less difficulty than solutions for T_2, T_3, etc. After the first inspection has taken place, all probabilities must be computed with the knowledge of system survival until time = T_{i-1}.

EXAMPLE 4

Consider the case where an inspection costs \$50 (A=50), a failure costs \$900 (B=900), the time to fail follows the Weibull distribution with a shape parameter of 1, a scale parameter of 200 days, and a location parameter of 0. The lapse distribution is Weibull with a shape parameter of 5, a scale parameter of 30, and a location parameter of 10.

Solution

Since the shape parameter of the Weibull distribution is equal to 1.0 for the time to fail distribution, the time between each successive inspection will be constant. The optimum point is shown graphically in the figure below.

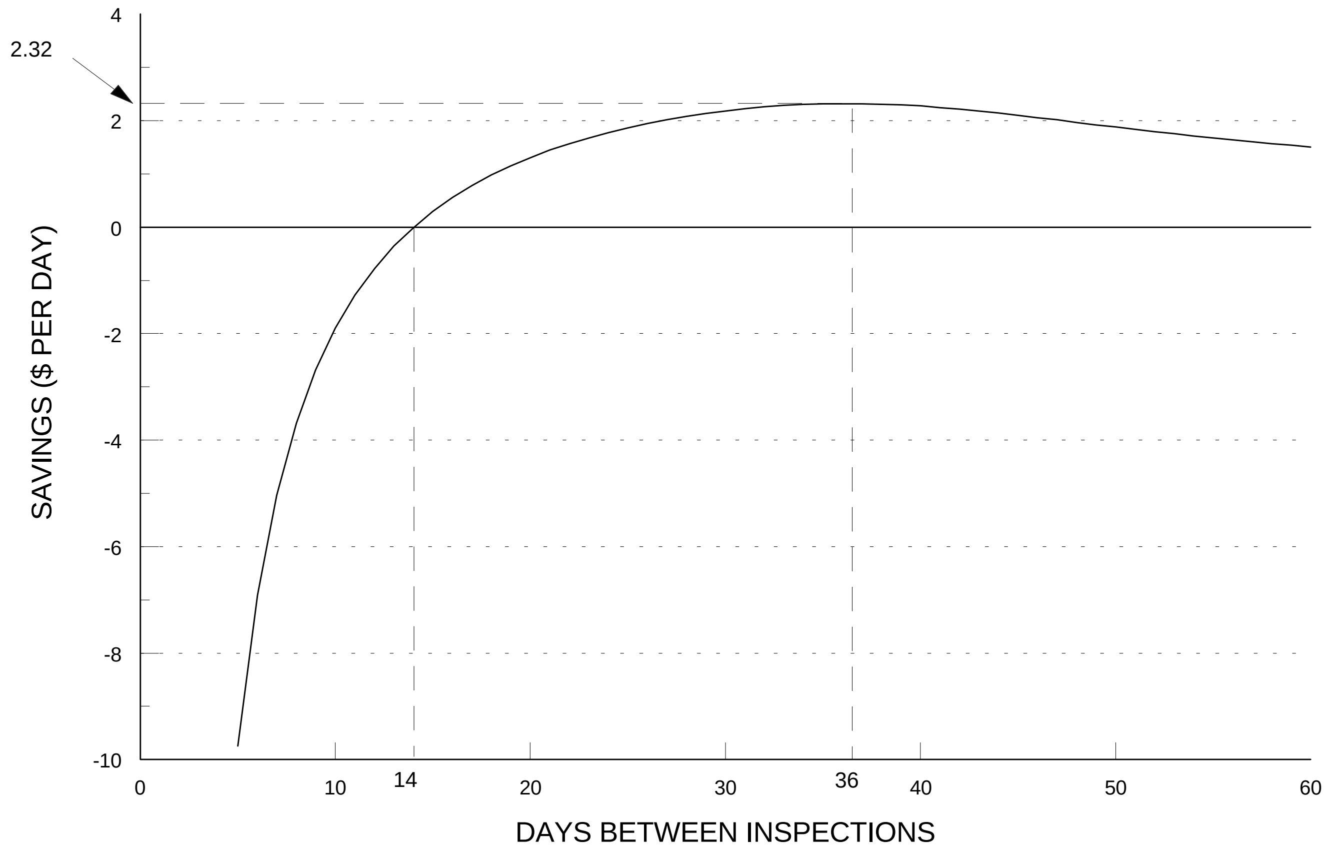

Figure 7. Optimum schedule for predictive maintenance.

EXAMPLE 5

Suppose the shape parameter for the time to fail distribution in Example 4 were changed from 1.0 (a constant failure rate) to 2.5 (an increasing failure rate). The optimum time for the first inspection is now 120 days as opposed to 36 days when the failure rate was constant. This increase is because the variance of the time to fail distribution is less for this example than for Example 4. The variance of the Weibull distribution decreases as the shape parameter increases. If there is no failure or detection of an impending failure, the optimum time for the second inspection is 152 days, and the third 182 days. Notice that the time between each successive inspection is decreasing.

CONCLUSIONS

Preventive maintenance, inspections, and predictive maintenance are all tools that can be used to reduce maintenance expenses. Like any tool, maximum gain is obtained by using these tools correctly. Significant financial gains can be made by collecting failure data and optimizing maintenance schedules.

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